The Evolving Universe and the Puzzling Cosmological Parameters

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ABSTRACT

The universe is found to have undergone several phases in which the gravitational constant had different behaviors. During some epochs the energy density of the universe remained constant and the universe remained static. In the radiation dominated epoch the radiation field satisfies the Stefan's formula while the scale factor varies linearly with time time. The model enhances the formation of the structure in the universe as observed today.

Introduction

The idea of a variable gravitational constant was first introduced by Dirac [4] in his Large Number Hypothesis, that the fundamental constants of physics and astronomy are related by some simple functions of our present epoch. He thus suggested that the gravitational constant G, as measured in atomic standards, varies with time as t^{-1} and the number of nucleons in the universe varies as t^2. While Dirac suggestion that $G \propto t^{-1}$ during the entire evolution of the universe, Brans-Dicke (BD) [5] theory suggests that the gravitational constant has a different evolution in different eras. In BD theory the gravitational constant is related to the inverse of a scalar field that changes with time. The Brans-Dicke theory represents an extension of the general theory relativity to include a scalar particles for the propagation of the gravitational interaction (long range). However, these particles have not yet been observed. For a positive coupling between scalars and geometry (ω), the theory implies a decreasing gravitation constant, though some application requires this parameter to be negative in th early universe [8,9]. At the present time observations require ω to be 500. Models of an increasing gravitational constant with a presence of a decaying cosmological constant are recently proposed by Abdel Rahman [2], Beesham [7]. In these models the variation of the gravitational constant is canceled out by the variation in the cosmological constant so that the energy conservation law remains intact. Though this approach is not covariant yet looks appealing. Bertolami [8] has found a similar solution in the BD theory for the matter dominated present epoch as well as the radiation dominated epoch, but with a negative ω. Abdel Rahman has shown that during the radiation dominated universe the gravitational constant varies quadratically with time. In

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the same line we have generalized this work to include a general variation of G with time in the presence of a cosmological term having $\Lambda \propto t^{-2}$ [12]. Thus our model encompasses Dirac's model as well as Brans-Dicke model. Brans-Dicke and Dirac models are said to be Machian as they satisfy Mach's principle [11]. It has been shown that the relation $\frac{GM}{Rc^2} \sim 1$ is valid for the whole evolution of the universe and represents the Mach's principle [11]. Employing this relation together with our model we have been able to discover what the early universe as well as the present universe looks like. In an earlier work we have shown that our G-varying model is consistent with the palaeontological as well as geophysical data so far known [13].
The unification recipe was applied to the three interactions excluding gravity. It depends on the idea that the coupling constants of the three interaction are running coupling constants (i.e. depend on energy). The three interactions get unified around the Grand Unified (GUT) scale where the temperature of the universe was around 10^{15}GeV. However, knowledge of the gravitational interactions at this epoch can only be extrapolated from the general theory of relativity. The idea of a variable gravitational constant can be viewed as making the gravitational coupling (G) a running coupling constant (i.e. G is made to depend on energy rather than time). Thus one may get some clue about the behavior of gravity in the very early universe that may not be possible in the framework of the ordinary general relativity. Though one believes that the appropriate way for a complete description of gravity near Planck's scale is the quantum gravity yet this approach, albeit, phenomenological merits some attention. The universe is found to evolve into different phases. The model is found to give a good account for the structure formation in the universe that we observe today.

Varying G and the Early Universe

In an earlier work, we have shown that $G \propto t^2$ and the scale factor $R \propto t$ in the radiation epoch [1]. Using Mach's principle ($c = 1$),

$$\frac{GM}{R} \sim 1 \tag{1}$$

We obtains

$$\frac{G_P}{G_N}\frac{M_P}{M_N} = \frac{R_P}{R_N}, \quad \frac{G_P}{G_G}\frac{M_P}{M_G} = \frac{R_P}{R_G}, \quad \frac{G_P}{G_W}\frac{M_P}{M_W} = \frac{R_P}{R_W}, \tag{2}$$

and

$$\frac{R_P}{R_N} = \frac{t_P}{t_N}, \quad \frac{R_P}{R_W} = \frac{t_P}{t_W}, \quad \frac{R_P}{R_G} = \frac{t_P}{t_G} \tag{3}$$

and

$$\frac{G_P}{G_N} = (\frac{t_P}{t_N})^2, \quad \frac{G_P}{G_W} = (\frac{t_P}{t_W})^2, \quad \frac{G_P}{G_G} = (\frac{t_P}{t_G})^2 \tag{4}$$

where, $\mathrm{M_W, M_G, M_N, M_P}$ are the masses at Electroweak ($\mathrm{t_W}$), GUT ($\mathrm{t_G}$), Nuclear ($\mathrm{t_N}$), and Planck's time ($\mathrm{t_P}$), and $\mathrm{R_G, R_W, R_P}$ are the radius of the universe at GUT, Electroweak and Planck time, respectively, $\mathrm{G_G, G_W, G_P}$ are the value of the gravitational constant at the GUT, Electroweak and Planck time, respectively. With $\mathrm{M_P} = 10^{-5}\mathrm{g}, \mathrm{R_P} = 10^{-33}\mathrm{cm}, \mathrm{t_P} = 10^{-43}\mathrm{s}, \mathrm{t_N} = 10^{-23}\mathrm{s}$, one obtains

$$G_P = G_0,\ G_N = 10^{40} G_0,\ G_G = 10^6 G_0,\ G_W = 10^{32} G_0, \tag{5}$$

$$M_N = 10^{-1}\text{GeV},\ \text{M}_\text{G} = 10^{16}\text{GeV},\ \text{M}_\text{W} = 10^{3}\text{GeV} \tag{6}$$

and

$$R_G = 10^{-30}\text{cm},\quad \text{R}_\text{W} = 10^{-17}\text{cm} \tag{7}$$

See Table (1) for a complete enumeration. According to the above equations one obtains

$$t_G = 10^{-40}s,\quad t_W = 10^{-27}s \tag{8}$$

Though the time for the GUT and Electroweak are different from the Standard Model the radius of the universe is the same at these epochs. A time $t = 10^{-35}$sec would correspond to a mass $M = 10^{11}$GeV. Notice that the relation $G \propto t^2$ is consistent with the above formula. In fact one observes that the Planck's mass is evolving with time. Thus one can say that the M_G, M_W, M_N are manifestation of this mass in different epochs of the evolution of the universe. Only at Planck's time the Planck's mass has the value $10^{-5}g$. The effect of the Planck's mass evolution is that gravitational strength changes with time too. And sine $G \propto t^2$, eq.(1) gives $M \propto t^{-1}$. This result shows that the matter is annihilated continuously as the universe expands. Thus it is clear that this annihilated mass will contribute to the increasing of the entropy.
It is clear that the universe in this model does not have a particle horizon or monopole problem, that exist in the standard model. We see that the gravitational constant had a very different (and big) value in the early universe. One also anticipates that the universe to enter an epoch characterized by

$$G \propto t^{-2},\quad R = \quad \text{const.},\quad \rho = \quad \text{const} \tag{9}$$

This solution is obtainable from BD theory with $\omega = -1$. It is generally known that this solution corresponds to a low energy string model. This period is essential to bring the value of the gravitational constant down to a reasonable value before nucleosynthesis starts. In this static universe the perturbations grows exponentially which will later become the seeds for formation of structure in the universe. In the matter dominated (MD) phase one has

$$G = G_0(\frac{t}{t_0}),\quad R = R_0(\frac{t}{t_0}) \tag{10}$$

where the subscript '0' denotes present day quantity. This variation law, together with eq.(1), gives, M=const. Thus the matter creation stopped by the advent of the matter domination. This situation continues up to the present epoch. From eq.(1) one can write

$$\frac{G_0}{G_P}\frac{M_0}{M_P} = \frac{R_0}{R_P},\quad \frac{G_0}{G_N}\frac{M_0}{M_N} = \frac{R_0}{R_N},\quad \frac{G_0}{G_W}\frac{M_0}{M_W} = \frac{R_0}{R_W},\quad \frac{G_0}{G_G}\frac{M_0}{M_G} = \frac{R_0}{R_G} \tag{11}$$

which give the same results as obtained from eqs.(5)-(8), and predict that the mass of the present universe is $\sim 10^{56}$g ! Thus we have shown that eq.(1) is very fundamental and gives us a lot of clues to our evolving universe.

Formation of galaxies

The development of inhomogeneities in the universe is described by the density contrast $\delta \equiv \frac{\delta\rho}{\rho}$. Since $\rho \propto R^{-4}$ in the radiation dominated (RD) phase ($t < t_{eq}$) and $\rho \propto R^{-3}$ in the matter

dominated (MD) phase ($t > t_{eq}$) we get (where t_{eq} is the time when matter and radiation were in equilibrium) [3]

$$\frac{\delta\rho}{\rho} \propto \left\{ \begin{array}{ll} R & t < t_{eq} \\ R & t > t_{eq} \end{array} \right\}$$

It clear that the density contrast is the same in both radiation dominated and matter dominated phase. The amplitude of mode with $\lambda > d_H$ always grows, where λ is the wavelength of the perturbation and d_H is the Hubble radius. The future growing modes after entering the Hubble radius($\lambda < d_H$) depend on the pressure distribution of matter which may prevent gravitational enhancement of the density contrast. This is characterized by the Jeans length λ_J where [3,6]

$$\lambda_J = \sqrt{\pi}\frac{v}{(G\rho)^{1/2}} \tag{12}$$

(v is the velocity dispersion of the perturbed component) and ρ is the density of the component which is most dominant gravitationally, i.e. the one that makes the perturbation collapse. For all $\lambda >> \lambda_J$ in the matter dominated phase, the amplitude grows as R. Note that $v \propto R^{-1}$ due to the red shifting of the momentum $p \propto mv \propto R^{-1}$. One can now define the Jeans's mass as [3,6]

$$M_J = \frac{4\pi}{3}\rho(\frac{\lambda_J}{2})^3 \tag{13}$$

Since $\rho \propto R^{-4}$ in the radiation dominated phase and $\rho \propto R^{-3}$ in the matter dominated phase, we see that

$$M_J \propto \left\{ \begin{array}{ll} R^{-1} & R < R_{eq} \\ R^{-3} & R > R_{eq} \end{array} \right\}$$

The mass inside the Hubble radius, M_H is similarly defined to be [3,6]

$$M_H = \frac{4\pi}{3}\rho(\frac{d_H}{2})^3 \propto \left\{ \begin{array}{ll} R^{-1} & R < R_{eq} \\ \text{const.} & R > R_{eq} \end{array} \right\}$$

In our present model M_H evolves as t^{-1} in the early universe and remains constant during the matter dominated epoch. While $M_H \propto t$ in the standard scenario, our prediction is that M_H is constant in the matter dominated phase. We see that in the radiation dominated phase $M_J \propto M_H \propto t^{-1}$, so if initially $M_J > M_H$ then no matter will be formed in the radiation dominated phase but if $M_J < M_H$ matter is likely to form in this phase. M_J decreases as R^{-3} (faster than the standard scenario) and $M_H = \text{const.}$ in the matter dominated phase. This is unlike the standard scenario where $M_J \propto M_H \propto t$ in the RD phase and $M_J \propto M_H^{-1} \propto t^{-1}$ in the MD phase. We remark that the present model is consistent with the CMBR since we have in the RD phase $T \propto R^{-1}$. In the standard scenario the Jeans mass at equilibrium between radiation and matter is calculated to be [10]

$$M_J^s = \frac{4\pi}{3}(\frac{T_r}{Gm_H})^{3/2}\frac{1}{\rho_{eq.}^{1/2}} \sim 10^5 \mathrm{M}_\odot \tag{14}$$

Where M_J^s is the standard model value, T_r the temperature of radiation at equilibrium, ρ_{eq} the density of matter at equilibrium, and m_H is the mass of hydrogen atom. In our scenario the Jeans mass is

$$M_J = (\frac{t_{eq}}{t_0})^{3/2} M_J^s \sim 10^{11} \mathrm{M}_\odot \tag{15}$$

where we have used $t_{eq} = 10^6$ year. This is a typical mass of a galaxy. The mass in the horizon radius, M_H at the equilibrium is given by, with eq.(10),

$$M_H = \frac{c^3 t_{eq}}{G_{eq}} = \frac{c^3 t_0}{G_0} \sim 10^{11}\mathrm{M}_\odot \tag{16}$$

Thus we see that $M_J \sim M_H$ at equilibrium. We also notice that M_H is independent of time in the matter dominated phase. Thus this mass is frozen on the onset of the recombination time which is unlike the Jeans mass which still decreases with time in both eras. This may show that why we don't observe galaxies with masses much bigger than this. In fact, this mass is of the same order of the mass considered from damping of photon viscosity, which was found to vary as $T^{-9/2}$ [10]. Thus a more quantitative analysis should be done before one decides to take this model as an alternative to the standard scenario. In fact our model behaves like a universe filled with string-like matter with an equation of state $p = -\frac{1}{3}\rho$ and with G constant!

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Table 1: Cosmological Parameters of the very Early Universe

Parameter	Planck(P)	GUT(G)	Electroweak(W)	Nuclear(N)	Present(0)
Time	$10^{-43}s$	$10^{-40}s$	$10^{-27}s$	$10^{-23}s$	$10^{17}s$
Radius of the universe	10^{-33}cm	10^{-30}cm	10^{-17}cm	10^{-13}cm	10^{28}cm
Strength of gravity	G_0	$10^6 G_0$	$10^{32} G_0$	$10^{40} G_0$	G_0
Density of the universe	10^{94}gcm^{-3}	10^{62}gcm^{-3}	10^{26}gcm^{-3}	10^{14}gcm^{-3}	10^{-29}gcm^{-3}
Mass Scale	10^{19}GeV	10^{16}GeV	10^{3}GeV	10^{-1}GeV	10^{80}GeV